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Contrails and aircraft downwash

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Aircraft downwash consists initially of a vortex pair descending with its accompanying fluid through the atmosphere. Condensation trails are formed in exhaust emitted into the accompanying fluid and the shapes of them and their evolution depend on the positions of the engines in relation to the wing tip vortices.

The atmosphere is stably stratified and so the descending accompanying fluid acquires upward buoyancy. Consequently vorticity is generated at the outside of the accompanying fluid and the flow pattern in the vortex pair is altered so as to produce detrainment of its exterior part. So long as any air which is a mixture of accompanying fluid and exterior air is detrained, the vortices remain stable, but the width of the pair decreases and its downward velocity increases with time as a result of the buoyancy. Eventually the upper stagnation point in the motion relative to the vortices begins to move upwards relative to the vortices so that some mixed fluid is entrained into the circulation and the vortices immediately become unstable, mixing occurs, the pressure in the core rises, and any vortex core trails that may exist appear to burst.

The motion produces downward-thrust blobs in trails from centrally placed engines, which correspond to the holes sometimes seen in cloud when distrails are formed.

1. Introduction

Contrails are clouds of water droplets or ice particles formed in the exhaust of aircraft. The appearance of these trails is determined by the physics of the condensation, freezing, and evaporation mechanisms and by the motion to which they are subjected. In order to interpret their shapes in terms of this motion which is the aircraft downwash, it is necessary to understand the physics, and a summary of this is given first.

In 1955 Scorer described the appearance of contrails in terms of the motion of the vortex pair which is the aircraft downwash. It was then suggested that the loops which appear were a manifestation of the instability inherent in any vortex pair in which the nearness of the vortices is slightly varying, but no quantitive theory was given. In 1957 Turner described the effect of buoyancy on vortex rings, and the implications of this were included in Scorer's (1958, pp. 73–4 and plate 2) description of contrail forms. It was appreciated that if, anywhere, the circulation round a vortex decreases outwards the motion is unstable and the buoyancy forces in the accompanying fluid obviously operate to produce this effect because the circulation in a cylindrical thermal would be opposite to that in a vortex pair moving downwards. The effect of the buoyancy is to produce detrainment of accompanying fluid, because it decreases the distance between the vortices according to Turner's theory. The result according to the present investigation is that to begin with all the opposite vorticity generated is detrained and so no instability is produced in the vortices, but after a time which depends almost entirely on the static stability of the air, instability will occur and the vortices will suddenly burst.

The consequences of these developments are clearly visible in contrails and distrails (clear lanes in cloud made by the downwash), and a comprehensive collection of pictures of them has been made by Scorer (1970). The results of the present paper are taken from Davenport (1967).

2. Contrail physics

In figures 1–3 the curves give the saturation ratio in g kg^{-1} of water vapour to air as a function of temperature for three different pressures. The arrow marked 'exhaust' indicates the direction of the point representing typical

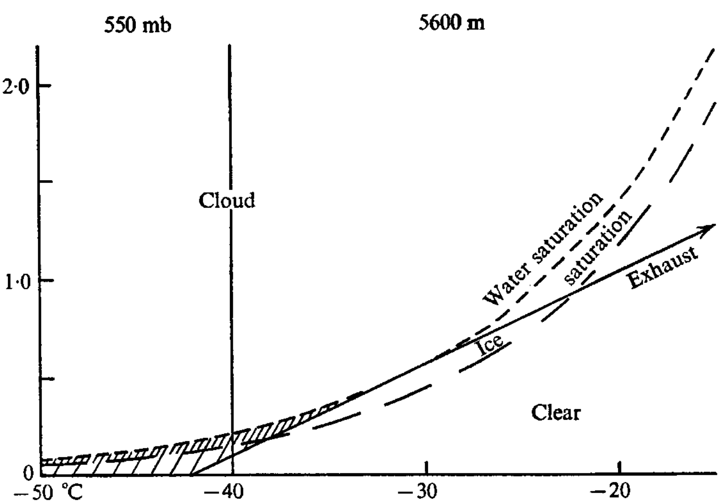

FIGURE 1. Water and ice saturation mixing ratio curves for air at 550 mb. If the point representing the ambient air lies in the shaded region a contrail will form as the exhaust, represented by a point in the direction indicated, is diluted isobarically. If the ambient air point is in the doubly shaded region and the trail is glaciated it will persist. It will be glaciated immediately if it reaches a temperature below −40 °C.

aircraft exhaust. As the exhaust mixes with the air, the mixture is represented by a point on the straight line joining the points representing the ambient air and the exhaust. As the mixing proceeds cloud will appear if water saturation is reached, which means that for a contrail to occur in previously clear air the ambient air point must lie within the shaded area below the water saturation line and above the tangent to it from the exhaust point.

If the cloud freezes it will not evaporate if the ambient air point lies in the doubly shaded area which lies above the ice saturation mixing ratio curve. If the ambient air is at a temperature below −40 °C the cloud will certainly freeze and in the above situation persistent contrails will occur. The shaded areas thus give where the ambient point must lie for the occurrence and persistence of trails respectively, if they are formed by mixing at constant pressure.

Figure 4 summarizes these conclusions and shows where in the atmosphere contrails of different kinds are likely to be found. The 'Mintra' is the boundary

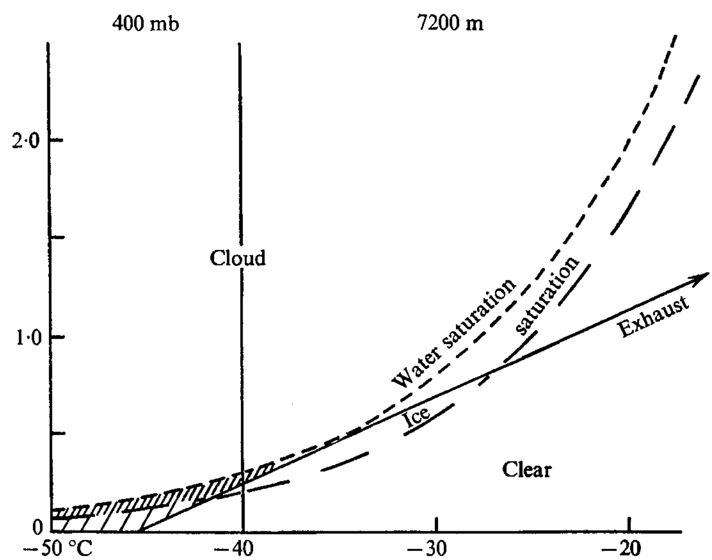

FIGURE 2. The same as figure 1 for 400 mb.

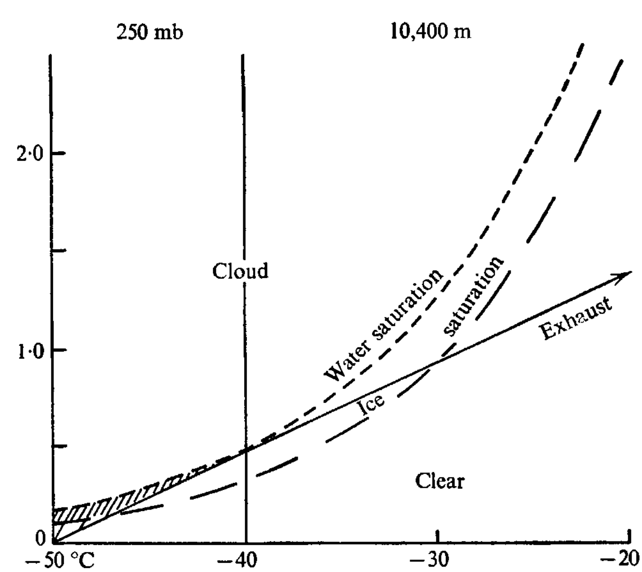

FIGURE 3. The same as figure 1 for 250 mb.

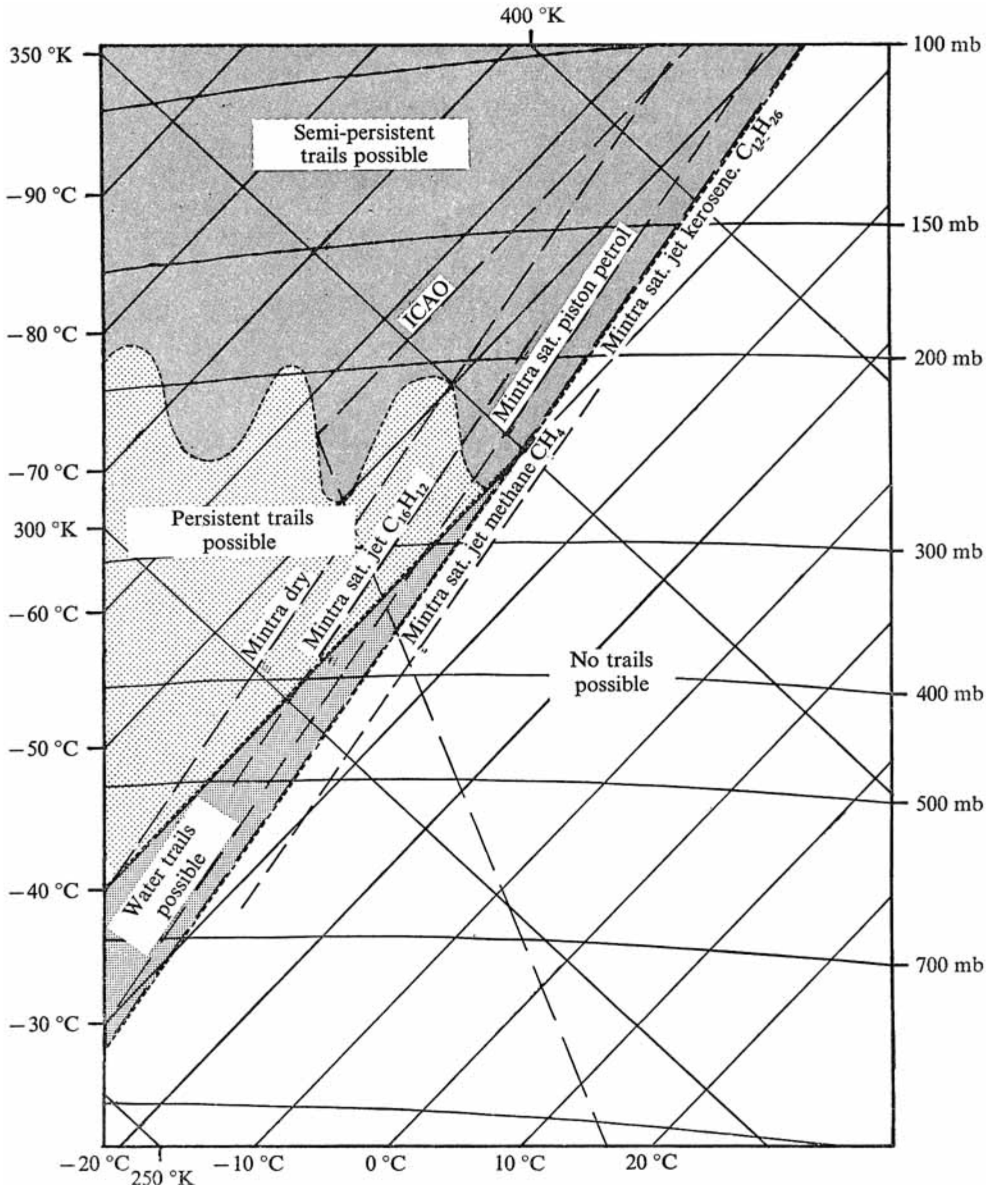

FIGURE 4. The conditions in the atmosphere required for condensation trail formation are shown on this T, ϕ diagram in which the two orthogonal co-ordinates are temperature (in °C) and log (potential temperature), the potential temperature being indicated in °K. The isobars are slightly curved and the diagram is drawn so that they are almost horizontal. Pressure and temperature are then used to plot a sounding of the atmosphere on the diagram: and the line marked ICAO represents the ICAO standard atmosphere, with a tropopause at about 230 mb.

The shaded areas are where trails are possible from jet aircraft burning kerosene: they are bounded by the 'Mintra' which indicates the pressure, at each temperature, below the altitude of which trails cannot be formed. The 'Mintra dry' indicates the limit for an absolutely dry atmosphere.

(For a description of the T, ϕ diagram see, for example, Scorer (1958, pp. 256–60).)

between where no trails are possible, and where they are possible. In the latter region they may remain unfrozen if the temperature is above -40 °C, and whether they persist or not if they are frozen depends on the humidity of the environment. Generally unfrozen trails are non-persistent because dilution always produces evaporation ultimately. They are called semi-persistent if they are frozen but the environment is not saturated for ice, because ice clouds usually take much longer to evaporate. In the stratosphere they are never persistent because of the low humidity, and so the upper boundary of the region where persistent trails would occur if the air were saturated with respect to ice is rather variable, and depends on where the tropopause is. This is indicated by the wavy boundary in the region of 250 mb. It is usually around 240 mb in the middle latitudes but can be quite a lot higher or lower.

The ICAO standard atmosphere is shown, to indicate the rough whereabouts of the points of a typical sounding: obviously an actual sounding can depart very much from this, but it indicates that the region where trails are most likely to be persistent is below the tropopause and above about 300 mb (9000 m very roughly).

The position of the 'Mintra' line depends on the nature of the aircraft exhaust, and to show its variability its position is given for a typical jet engine burning kerosene when the air is saturated for water and when it is absolutely dry. The manner in which its position might vary in saturated air is shown by examples for other hydrocarbon fuels which produce different amounts of heat and water vapour.

Vortex trails are not indicated in the diagram. They occur when exhaust enters the core of the wingtip vortices, where the pressure may be 50–100 mb lower, and they can occur when contrails are not formed by mixing, but the engine must usually be rather near to the wingtip. If vortex trails freeze they will persist if the ambient air is supersaturated for ice.

3. Motion due to a vortex pair

The variation along the length of an aircraft wake and the longitudinal component of velocity are so small that any section can be adequately represented as the same as that of an infinitely long two-dimensional vortex pair, and in the next section our analysis is like that of Turner (1960) who described the accelerations and changes in size of vortex rings and vortex pairs possessing buoyancy.

The streamlines relative to the vortices are shown in figure 5 for the case where the vorticity is concentrated into two thin lines. In practice it is diffused through a central core, but when the vorticity is not concentrated the velocity field outside the core, and more particularly the shape of the boundary of the accompanying fluid, is virtually unaltered.

Air circulates within this boundary and travels along with the vortices. The exterior air passes round the outside as the accompanying fluid descends with a speed which is equal to the speed of each vortex in the field of motion of the other.

In the right half we show the tracks along which narrow exhaust trails would circulate round the vortices. A trail from a centrally placed engine would simply descend vertically. In the left half we show the positions of three particles, originally placed along the line of the wing, at three successive times, and we see that the line of particles joining them becomes wound up. This means that

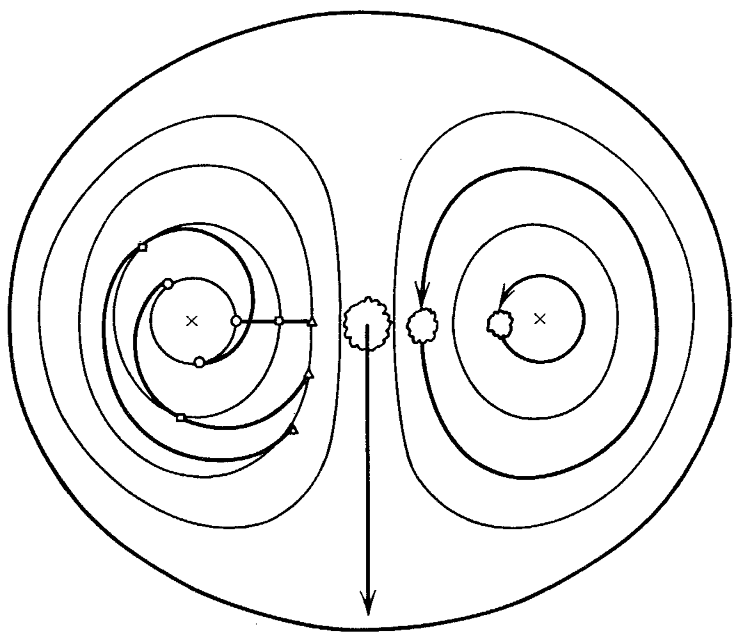

FIGURE 5. The streamlines in the motion due to a vortex pair located at the points ×. The trajectories of air parcels and lines of marked fluid particles in this motion pattern are indicated.

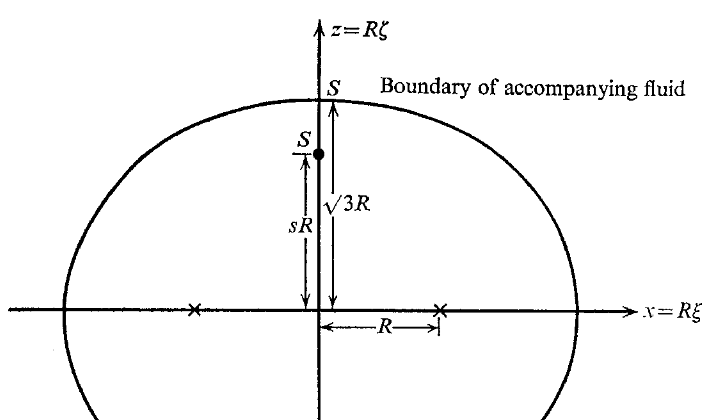

FIGURE 6. Co-ordinate system for a vortex pair.

although a trail near the wingtip circulates around the vortex rapidly, a trail nearer the centre of the wing will appear from a distance to circulate around the outer one. This is evident in the first photograph in figure 10, plate 1.

4. The effect of atmospheric stratification

If $2R$ is the distance between two horizontal line vortices at the same height Z, with circulations $\pm K$, their vertical velocity is given by

$$W = dZ/dt = -K/4\pi R. \tag{1}$$

The impulse required to establish the motion of unit length of the vortex pair in fluid density ρ (see, for example, Lamb 1932, art. 152, equation 6, or Davenport 1967) is

$$Q = -2\rho KR. \tag{2}$$

The front and rear stagnation points (L and S in figure 7) form equilateral triangles of side $2R$ with the vortices. The accompanying fluid, whose boundary passes through these points, is composed of air from the original level of the vortices, which is the flight level of the aircraft. If the environmental potential temperature is τ^{-1}, and has value τ_1^{-1} at the flight level, and

$$\beta = -\frac{1}{\tau_1}\frac{\partial \tau}{\partial z}, \tag{3}$$

the potential temperature anomaly of the accompanying fluid is given by

$$\Delta\tau/\tau_1 = \beta Z, \tag{4}$$

when the flight level is at $Z = 0$. In the atmosphere τ plays the same role as the density in an incompressible stratified fluid.

We may equate the rate of change of the impulse of the vortex pair at any moment with the total buoyancy force which results from the vertical displacement of the accompanying fluid in the stratified environment. Thus

$$dQ/dt = gmR^2\Delta\rho, \tag{5}$$

where $\Delta\rho$ is the density anomaly and is equal to $\rho\Delta\tau/\tau_1$, and mR^2 is the cross-section area of the accompanying fluid. This leads to

$$2K\,dR/dt = gmR^2\beta Z, \tag{6}$$

which by (1) gives

$$dR/dZ = -(2\pi gm\beta/K^2)R^3Z, \tag{7}$$

from which we obtain

$$R = R_1(1+R_1^2Z^2/h^4)^{-\frac{1}{2}}, \tag{8}$$

$$W = W_1(1+R_1^2Z^2/h^4)^{\frac{1}{2}}, \tag{9}$$

where

$$h^4 = K^2/2\pi mg\beta. \tag{10}$$

When expressed as functions of time the values of the height, size, and vertical speed of the vortex pair are given by

$$Z = \frac{h^2}{R_1}\sinh\frac{Kt}{4\pi h^2}, \tag{11}$$

$$R = R_1 \operatorname{sech} \frac{Kt}{4\pi h^2}, \tag{12}$$

$$W = W_1 \cosh \frac{Kt}{4\pi h^2}. \tag{13}$$

It is important to note later that the coefficient of t, namely $K/4\pi h^2$, does not actually depend on K.

5. Vorticity generation and detrainment

When the accompanying fluid is displaced downwards there is a density discontinuity at its boundary. We assume this discontinuity to have the same magnitude all over and the motion in the stratified environment to be approximately the same as when there is no stratification. This means that we assume the vorticity to be generated only at the boundary and not at all in the exterior fluid as a result of displacement in it. Meanwhile the vortices in the cores are assumed to retain a constant circulation because they are surrounded by fluid of uniform density in irrotational motion. These assumptions are justified when β, the stratification of the environment, is small and the density anomaly is changing slowly.

The rate of generation of vorticity at the boundary is given by

$$\frac{D\eta}{Dt} = \frac{\Delta\tau}{\tau}(g \sin\psi - f \sin\theta), \tag{14}$$

where ψ and θ are the angles made by gravity and the fluid acceleration vectors with the normal to the boundary, which is a streamline relative to the vortices. η is the vorticity density in the boundary and f is the magnitude of the fluid acceleration. This equation is simply the appropriate component of the vorticity equation applied to the thin layer of fluid which constitutes the boundary.

From equation (14) it is a routine matter to obtain the value of the vortex sheet strength η at all points on the boundary assuming it is zero at the lower stagnation point, for, with the co-ordinates of figure 7

$$\frac{D\eta}{Dt} = u\frac{\partial\eta}{\partial x} + w\frac{\partial\eta}{\partial z} \tag{15}$$

and the velocity components u and w are assumed to be those due to the vortex pair. The details are described in Davenport (1967). It emerges in the computation that the contribution due to the term in f is negligible except in circumstances mentioned below, and can usually be ignored in the case of the atmosphere.

When the vorticity has been calculated the velocity field may be recalculated. In particular the position of the upper stagnation point may be determined. It is now found to be inside the original boundary when the buoyancy is upward. Consequently when it is at S_1 in figure 7, for example, the fluid between the boundary and the streamline arriving at S_1 is detrained as a narrow strip of fluid left behind the system in the region bounded by the heavy line on the right side of the figure.

We suppose that the vortex layer on the boundary is dynamically unstable to the extent of producing corrugations or even mixing. The region pervaded by the density fluctuations will be confined to some region such as that shaded in figure 7. In the right half, when stagnation occurs at S_1 all the mixed fluid is detrained; but in the left half it is seen that, when stagnation is close to the original point, mixed fluid may be entrained between the vortices. In that case the motion

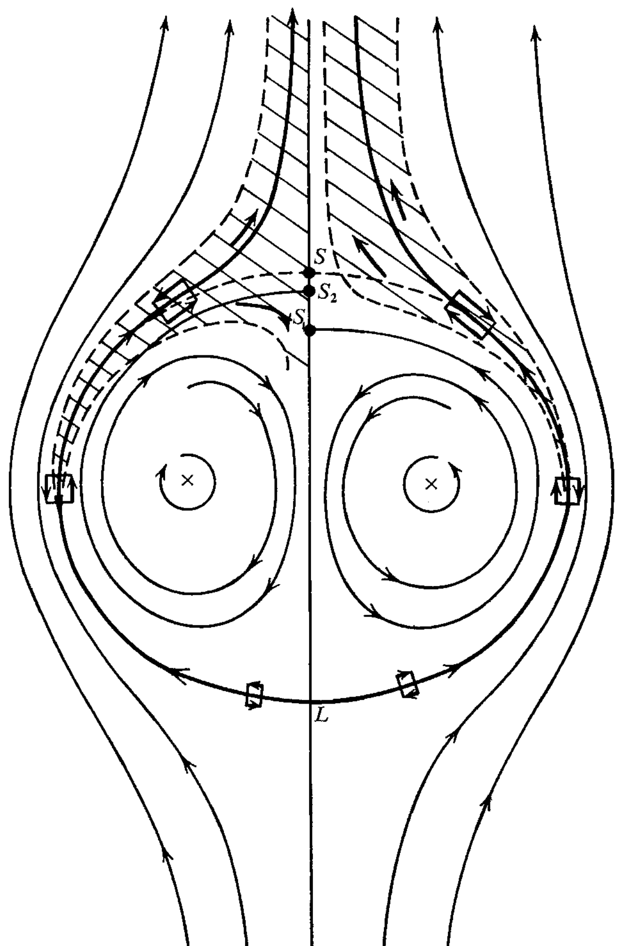

FIGURE 7. Streamlines when the accompanying fluid has buoyancy. The arrows on the boundary indicate the sense and magnitude of the vorticity generated by buoyancy. The shaded area represents the mixed region, and in the right half the upper stagnation point is at S_1 and the mixed fluid is detrained, whereas in the left half the pattern is shown where the stagnation point is in the mixed fluid, some of which is then entrained into the accompanying fluid.

inside the accompanying fluid would become dynamically unstable because the vorticity entrained would produce a decrease of circulation across the streamlines outwards from the vortices (without an alteration of the sign of the curvature). This is presumed to lead to a breakdown of the vortices.

Earlier theories of the mechanism of breakdown exist. Thus Squire (1954) supposed that there was an eddy viscosity proportional to the circulation, for this has the right dimensions. Its magnitude has to be determined from observation and Rose & Dee (1965) found that an explanation of the decay of the vortex cores in the early stages could be obtained in this way but not the subsequent very rapid breakdown. Owen (1964) proposed alternatively that the eddy viscosity might be proportional to the square root of the circulation. On either Squire's or Owen's theories the time of decay depends on the square root or fourth root of K, but the observations of Kerr & Dee (1960) suggested that the time was independent of the circulation, although this conclusion could not be drawn firmly in view of the inaccuracies inevitable in the measurements.

To us there seems no evidence of an eddy viscosity operating and some contrails in vortex cores remain very sharp for long periods. The motion is dynamically stable in the absence of an axial component of velocity, and it is very unsatisfactory to have to determine the eddy coefficient from the observations being explained.

6. The motion of the upper stagnation point

The vorticity on the boundary is given by

$$\eta = (2\pi R^2 g\beta Z/K)\phi, \tag{16}$$

where ϕ is the non-dimensional vorticity computed from

$$\partial\phi/\partial\xi = w/uq, \quad \partial\phi/\partial\zeta = 1/q, \tag{17}$$

which are derived from (14) and (15) by writing

$$x = \xi R, \quad z = \zeta R, \quad q^2 = u^2 + w^2, \tag{18}$$

and ignoring the term in f. Table 1 gives some of the values of ϕ obtained.

ξ	ζ	$\phi(\xi, -\zeta)$	$\phi(\xi, \zeta)$
0	1·73	0	5·42
0·42	1·70	0·37	5·05
0·83	1·59	0·75	4·66
1·25	1·39	1·17	4·25
1·67	1·05	1·66	3·76
1·92	0·69	2·05	3·36
2·07	0·23	2·50	2·92

TABLE 1. Values of the dimensionless vorticity at selected points on the boundary of the accompanying fluid

According to (16) the actual vorticity varies inversely as K, because the larger the circulation the more rapidly the fluid passes around the boundary, and so less vorticity is generated during its passage. The buoyancy represented by βZ increases downwards, but at the same time R decreases downwards according to (7). The consequence of these variations is that the vorticity at the boundary reaches a maximum relative to K and then decreases again. The stagnation point therefore descends from S as the system descends from its starting point of zero buoyancy, and then rises asymptotically towards S again.

If w_s is the vertical velocity produced by the non-dimensional vorticity ϕ on the boundary at the position $\xi = 0$, $\zeta = s$, the value of s at a stagnation point is given by

$$\frac{K}{\pi R(1+s^2)} - \frac{R^2 g\beta Z}{K} w_s = \frac{K}{4\pi R}, \tag{19}$$

this being the condition that the velocity there is the same as the velocity of the vortices. This can be written

$$\frac{3-s^2}{1+s^2}\frac{1}{w_s} = \frac{4\pi R^3 g\beta Z}{K}. \tag{20}$$

The right-hand side is negative and changes from zero to a maximum magnitude and then decreases exponentially with time in a manner determined by (11), and

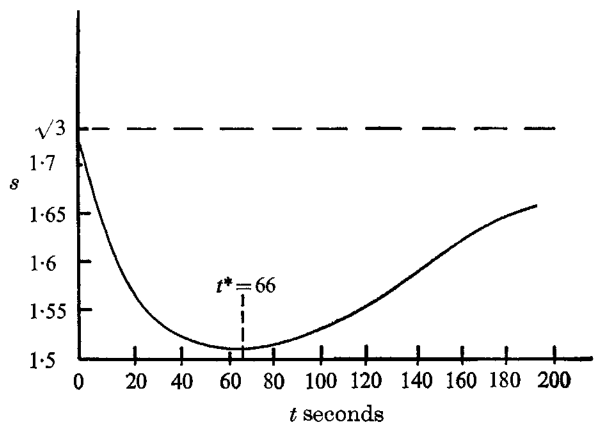

FIGURE 8. Changes in s with time for a typical atmospheric stratification. The stagnation point reaches its lowest position relative to the vortices after 66 sec and within another minute or two vortex core bursting can be expected.

(12). This is why s has a minimum at a certain time, which is independent of K, and is equal to

$$t^* = 0{\cdot}66(mg\beta/8\pi)^{-\frac{1}{2}}. \tag{21}$$

The value of m is 11·5 and so if $g = 10^3\,\text{cm sec}^{-2}$ we obtain

$$t^* = 98(\beta \times 10^7)^{-\frac{1}{2}}\,\text{sec} \tag{22}$$

if β is measured in cm^{-1}. With a typical value of β for the atmosphere in the range 10^{-6} to $10^{-7}\,\text{cm}^{-1}$ we see that t^* would be between about 30 and 100 sec, and is independent of K and R.

The value of s at its minimum is not so readily calculated because it depends on all the parameters of the problem. Its variation with time is shown in figure 8 for the following values

$$R_1 = 13\,\text{metres}, \quad K = 190\,\text{m}^2\,\text{sec}^{-1}$$

which corresponds roughly to a Comet aircraft, and

$$\beta = 2{\cdot}2 \times 10^{-7}\,\text{cm}^{-1}, \quad g = 10^3\,\text{cm sec}^{-2}$$

for a typical atmosphere.

Although the stagnation point returns towards its initial position rather slowly the intensity of the boundary vorticity is increasing and the dimensions of the accompanying fluid are decreasing. Consequently it is inevitable that fluid mixed at the boundary will become entrained into the accompanying fluid eventually, and that the vortex cores will then become unstable and rapidly mixed with a consequent rapid rise in central pressure and disappearance of any tip vortex trails that might have existed.

In the above calculation the term in f in (14) was ignored. This is justified in the example taken for $t < 3t^*$. This condition is not satisfied when the dimension R becomes so small that the fluid acceleration becomes comparable with gravity, with constant K. The effect is merely to retard the return of the stagnation point towards its original position.

7. The form of the instability of the vortex pair

The vortex cores can be seen to 'burst' when tip contrails are present. According to (11) or (13) the vortex pair is seen to be carried downwards with a constant velocity at first, but later with an exponentially increasing velocity. This means that in the later stages, though not at first, irregularities in the original vortex pair will grow exponentially and the vortices will assume an undulated configuration as shown in figure 9 with the width of the pair decreased

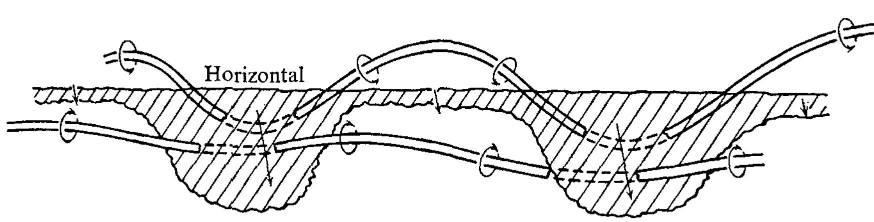

FIGURE 9. Perspective diagram of the relative positions of the cloud blobs and the burst parts of the tip vortices in the aircraft wake.

at the troughs as compared with the crests. Furthermore, because of the exponential growth, the configuration will be in a series of arches without significant regions of upward curvature. The tip trails will burst at the bottoms of the troughs. When bursting begins it will spread along the tip vortices as they descend.

If there is any cloud from a nearly centrally placed engine it will be detrained at the top if it has had time to pass round the vortices leaving a straight upper edge, while the bottom of it will be caused to descend in a series of blobs (figure 9) which correspond to where the vortices are closer together at the troughs.

8. Interpretation of contrails

The viewpoint of the observer is important. In figure 10(*a*), plate 1, the two trails from the starboard engines appear to separate, while those from the port engines seem to merge. This is simply because the inner trails are rotating around the outer ones. Soon after, a little further behind the aircraft, the fuzziness of the outside of the trails vanishes because the trail is unfrozen and mixing causes evaporation; but the tubular trails in the lower pressure regions of the vortex cores remain, and the vortex motion, being stable, becomes laminar away from the turbulence close to the aircraft. Evidently in this case exhaust did not penetrate into the vortex centres. In figures 10(*c*) and (*d*) the irregularities grow and finally mixing occurs and the cloud disappears.

By contrast figure 11, plate 2 shows the configuration of the trail of centrally placed engines. The curtain of detrained cloud has the vortex pair below it which produces a row of blobs in the cloud as it breaks up. This trail is frozen and persistent, while in figure 12, plate 2, the detrained cloud is scarcely visible and we see loops in the vortices rather than blobs in the cloud.

In figure 13, plate 3, we see artificial smoke trails emitted at 10,000 ft. from the wingtips of a Comet aircraft showing the same evolution with time, photographed from directly below. They are much narrower because they are emitted at the wingtips. The linking of the two ends where breaks occur is fairly typical also in contrails when the cloud is dense enough, but it is beyond the scope of this paper to discuss it.

Finally, the downwash of aircraft sometimes makes a clear lane (distrail) in a thin layer of cloud. Very occasionally the height of the aircraft above the cloud is such that a row of holes rather than a clear lane is made. This is illustrated in figure 14, plate 4.

The times and configurations of contrails are thus seen to fit the explanation suggested here fairly well but we have not predicted any particular space for the blobs which are produced. Irregularities which disturb both vortices together would grow more than those which only disturb one vortex. Consequently it is to be expected that irregularities with a space rather larger than the distance between the vortices would probably appear. If their spacing were too large there could well be another disturbance growing between two widely spaced irregularities. The pictures show that the spacing is typically three or four times the distance between the vortices, but that there is no clearly dominant wavelength. Probably it would be necessary to have an absolutely calm atmosphere for a dominant wavelength to appear as such.

We are indebted to Mr J. B. Holliday, of Rolls Royce Limited, Derby, for information about engine exhaust used in figure 4, to Mr F. W. Dee for the photographs in figure 13 which were first shown in a paper by Dee & Nicholas (1969), and to the Meteorological Office for their sponsorship of the work.

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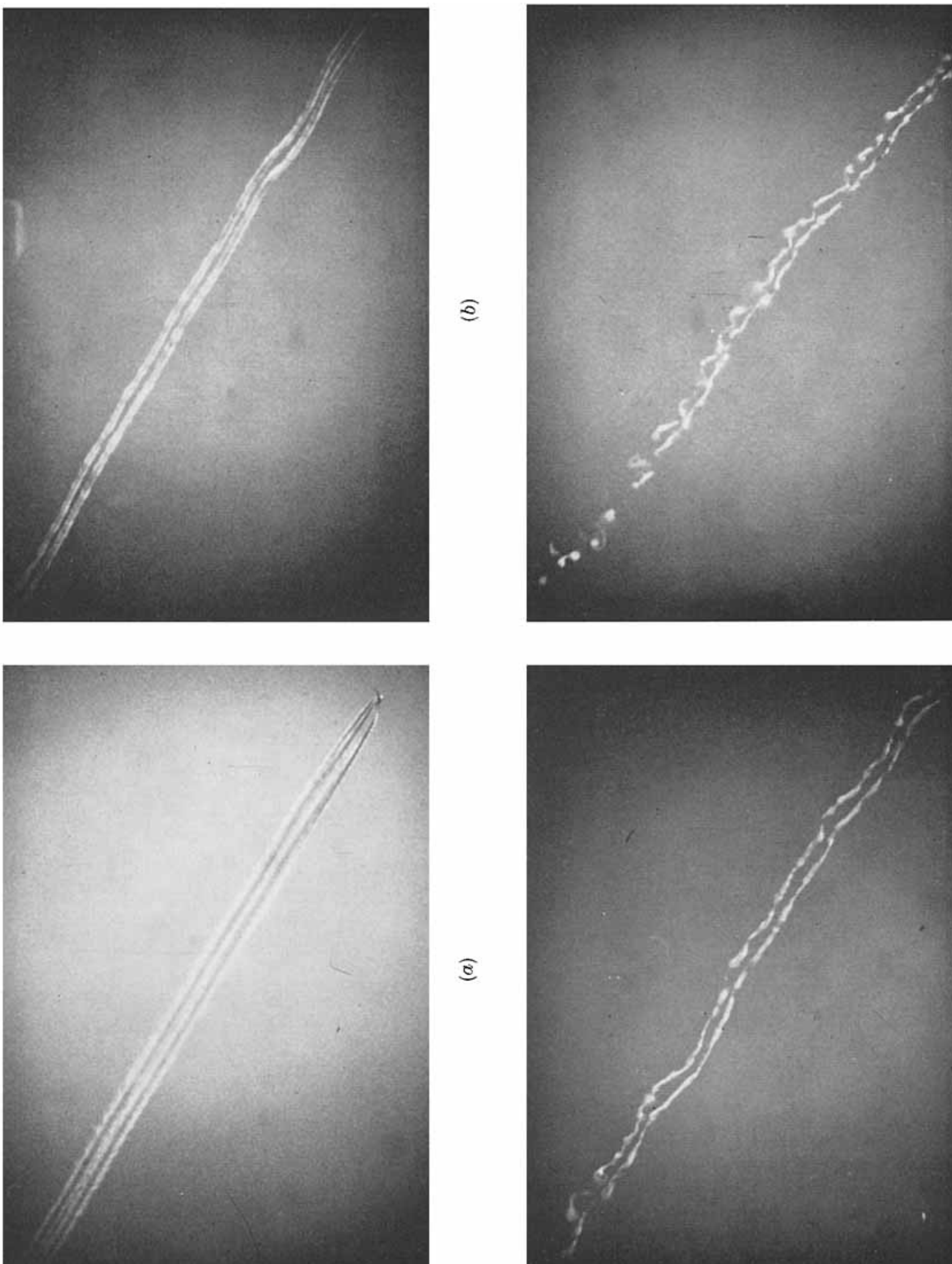

FIGURE 10. Development over about 2 min of a trail from a four-engined aircraft. The trail is not frozen and evaporates quickly except in the vortex cores.

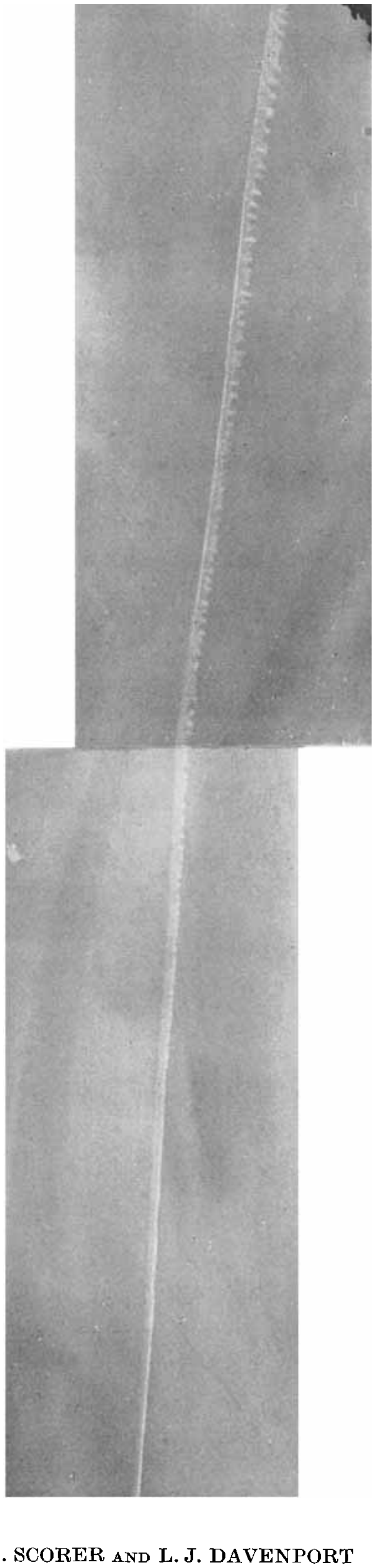

FIGURE 11. Two adjacent views of the same contrail showing the development with time.

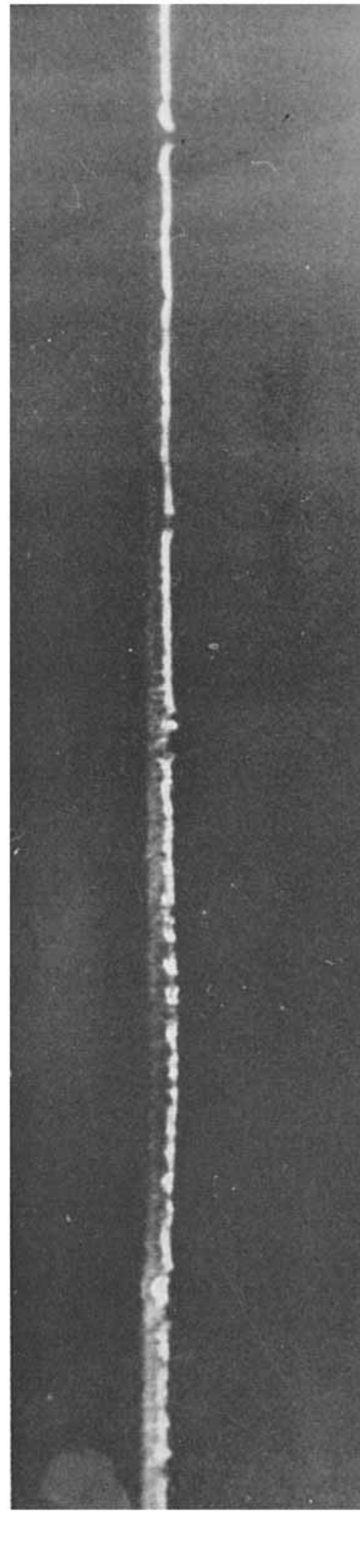

FIGURE 12. A trail similar to figure 11 but with the vortices rather than the curtain of cloud most clearly visible.

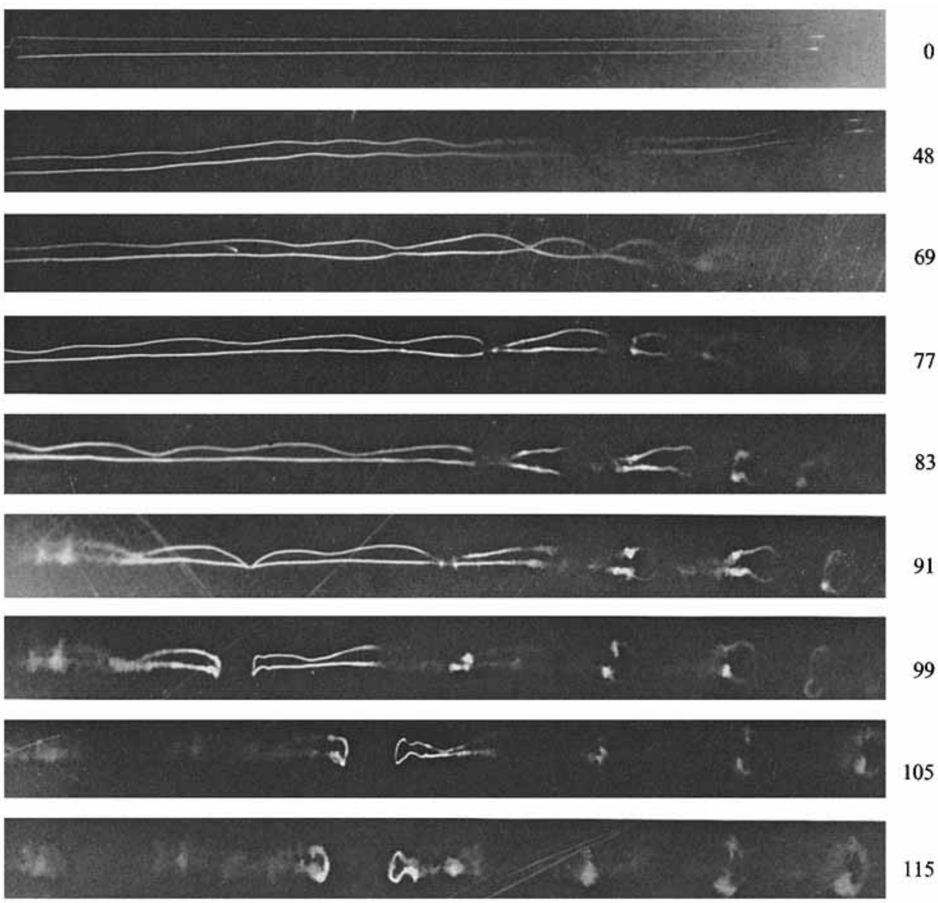

FIGURE 13. A series of ten pictures of smoke trails produced at the wingtips of a Comet aircraft at 10,000 ft., and photographed from below. The times are indicated in seconds.

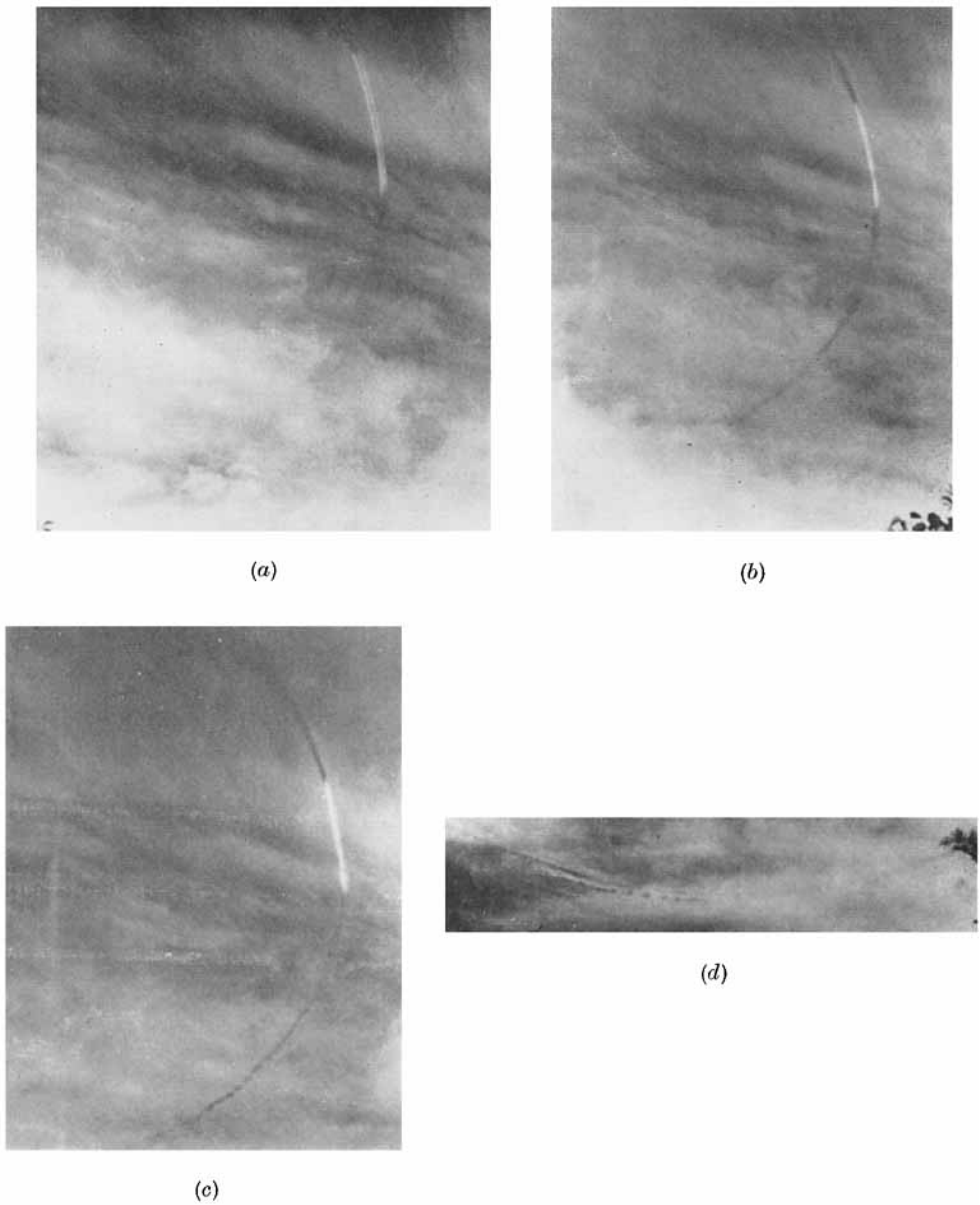

FIGURE 14. Four successive pictures of the effect produced by an aircraft descending into a thin layer of cloud while making a turn. In (*a*) the prominent feature is the dense trail where the aircraft is in an ice cloud. The distrail is just beginning to show.

In (*b*), (*c*) and (*d*) we see the subsequent development of the distrail, partly as a row of holes, corresponding to the trail blobs.